Big List Problems $\left(\mathbf{9}\frac{3}{4}\right)$:
On Uniform Continuity and Parametrizations

1. Find parametrizations (in either 1 or 2 variables, whichever is appropriate) of the following surfaces in $\mathbb{R}^3$: (i) The "double cone" obtained by rotating the line $y = x$ in the plane around the y-axis (ii) the intersection of the cone from part (i) with the sphere $x^2+y^2+z^2 = 1$ and (iii) an infinitely long cylinder of radius r.

2. (i) Find a continuous function $\gamma : \mathbb{R} \to \mathbb{R}^2$ whose image is the set formed by the circle of radius 1 along with the square with vertices $(\pm 1, \pm 1)$. (In other words, find a parametrization of this set). (ii) Describe the image of the following curve in $\mathbb{R}^3$: $\gamma(t) = (\cos t, \sin t, t)$

3. Show that if $f : A \to B$ is uniformly continuous on A, and $g : B \to C$ is uniformly continuous on B, then $g \circ f$ is uniformly continuous on A.

4. If f and g are uniformly continuous and bounded, show that fg is uniformly continuous. Find a counterexample to this statement if we omit of boundedness hypothesis.