Mini-Problems 7

1. Suppose that $w = x^2 y + e^{z+\sin(x)} + \log(x)$ and $x = e^t$, $y = \sqrt{t}$, $z = \log(t)$. Calculate dw/dt both by using the chain rule, and directly by writing w explicitly as a function of t and make sure your answers agree.

2. Let $f : \mathbb{R}^3 \to \mathbb{R}^2$, $g : \mathbb{R}^2 \to \mathbb{R}^2$ and $h : \mathbb{R}^2 \to \mathbb{R}^3$ be functions such that $g(x, y) = (x^3 y + 2y, 1 + xe^y)$, $f = (f_1, f_2)$ where $f_1(x, y, z) = 2x + 3y + 5z$, f_2 depends only on x, $\partial_1 f_2(1, 0, 1) = 3$, $f(1, 0, 1) = (0, 0)$ and

$$Dh_{(0,1)} = \begin{pmatrix} 0 & 1 \\ -1 & 3 \\ 0 & 1 \end{pmatrix}.$$

Compute $D(h \circ g \circ f)_{(1,0,1)}$ and the Jacobian of $h \circ g \circ f$ at $(1, 0, 1)$.

3. Let k be an integer. A function $f : \mathbb{R}^n \to \mathbb{R}$ is called homogeneous of degree k if $f(\lambda x) = \lambda^k f(x)$ for all $\lambda \in \mathbb{R}$ and $x \in \mathbb{R}^n$. Prove that if f is homogeneous of degree k then $x \cdot \nabla f(x) = kf(x)$.

4. Consider a function $f : \mathbb{R}^n \to \mathbb{R}$ and define $g(x_1, \dots, x_n) = f(x_1 - x_2, x_2 - x_3, \dots, x_n - x_1)$. Show that $\sum_{i=1}^n \partial_i g = 0$.