Mini-Problems 5

1. Suppose that $f : A \to B$ is bijective and continuous, where $A \subseteq \mathbb{R}^n$ is compact and $B \subseteq \mathbb{R}^m$. Prove that $f^{-1} : B \to A$ is also continuous.

2. Is the closure of a connected set connected? How about the boundary, or the interior? Give proofs or counterexamples.

3. Prove that if $A \subseteq \mathbb{R}^n$ is connected and open, then it is path connected. Hint: let $x_0 \in A$ be any point. Consider the set of points of A which can be reached from x_0 by a continuous path. Show that this set is both open and closed in A.

4. You know that a continuous function on a compact set attains its minimum and maximum. Prove the converse: if $A \subseteq \mathbb{R}^n$ is a subset such that every continuous function on A attains its maximum, then A is compact.