Mini-Problems 3

1. Consider the sequence (x_n) in $\mathbb{R}$ defined by $x_n = \sqrt{(n^2+n)} - n$. Find the limit and prove that (x_n) converges to it using the definition of the limit of a sequence.

2. Suppose that $(x_n)_{n=1}^{\infty}$ is a sequence in $\mathbb{R}$. Consider the following condition: for all $k \geq 0$, the sequence $(x_n - x_{n+k})_{n=1}^{\infty}$ converges to 0. Is this equivalent to saying that $(x_n)_{n=1}^{\infty}$ is Cauchy? Either prove this or provide a counterexample.

3. Let $(x_n)_{n=1}^{\infty}$ be a sequence in $\mathbb{R}^k$ such that $||x_m - x_{m+1}|| \leq 2^{-m}$ for all $m \geq 1$. Prove that (x_n) is Cauchy. (Compare this with the 4th exercise on Problem Set 3.)

4. Prove that if a sequence (x_n) in $\mathbb{R}$ converges, then so does the sequence $(|x_n|)$ (use the reverse triangle inequality). Is the converse statement true?