Mini Problems 20

1. Calculate the line integral of the vector field $F(x, y, z) = (y+z\cos(xz), 2y+x, x\cos(xz))$ along the path defined by the intersection of the cone $x^2+y^2 = z^2$, the plane $z = \frac{1}{2}x + 1$ and the half-space $x \geq 0$ oriented so that the tangent vector points in the positive y-direction at the point $(2, 0, 2)$.

2. Consider the vector field $F(x, y) = \frac{1}{x^2+y^2}(-y, x)$ defined on the set $\mathbb{R}^2 - \{(0, 0)\}$. In Example 5.18 of the notes, it is shown that F is closed but not conservative (and hence not exact) on this domain by calculating the integral of F around a circle centred at the origin. On the other hand, prove that

$$\int_\gamma F \cdot ds = \int_\Gamma F \cdot ds$$

for any two paths γ and Γ having the same endpoints and not intersecting the half-line $\{(x, 0) : x \leq 0\}$.

3. Suppose you have a funnel D, defined by the equations $x^2 + y^2 = z^2$ for $1 \leq z \leq 9$ and $x^2 + y^2 = 1$ for $0 \leq z \leq 1$, whose density at the point (x, y, z) is $\rho(x, y, z) = 16 - z$. Find the total mass of the funnel. (Sketch it first, so that you know what integral to write down.)

4. Calculate the flux of the vector field $(y, -x, z)$ through the funnel D of the previous problem, using outward-pointing normal vectors.