Mini-Problems 2

1. Prove that a set $A \subseteq \mathbb{R}^n$ is open if and only if it does not contain any of its boundary points.

2. Find the interior, the boundary, and the closure of each of the following sets X. Is X open, closed, or neither? **(i)** $X = \{\frac{1}{n} : \; n = 1, 2, \ldots\}$ inside $\mathbb{R}$ **(ii)** $X = \{(x, y) \in \mathbb{R}^2 : \; y = qx \text{ for some } q \in \mathbb{Q}\}$.

3. You have learned that two vectors $(a, b) \neq (0, 0)$ and $(c, d) \neq (0, 0)$ in $\mathbb{R}^2$ are orthogonal if $0 = (a, b) \cdot (c, d) = ac + bd$. Confirm that this is equivalent to saying that the lines defined by (a, b) and (c, d) (i.e. the two lines passing through the origin and each of these vectors) are perpendicular by directly calculating the condition for the latter.

4. Prove that $(\sum_{i=1}^n a_i)^2 \leq n \sum_{i=1}^n a_i^2$ for any real numbers $a_1, \ldots, a_n$. (Hint: use an inequality you learned in class.)