Mini Problems 19

1. Evaluate the line integral

$$\int_C y^3\,dx - x^3\,dy$$

where C is the curve defined in polar coordinates by $r = 1 - \sin(\theta)$.

2. Plot the curve $x(t) = (t^2 - t^3, t^3 - t)$ on a computer, and then find the area of the closed loop appearing in the curve. (For this problem and the next it may be helpful to remember the result of big list problem 19.5(1).)

3. Use Green's theorem to derive a formula for the area of a triangle whose corners are situated at the points (a_1, b_1), (a_2, b_2), (a_3, b_3) in counterclockwise order around the centre of the triangle.

4. Let D be a region of the plane for which the hypotheses of Green's theorem are valid. Show that Green's theorem may be recast as the formula

$$\int_{\partial D} F \cdot \hat{n}\,ds = \int\int_D \nabla \cdot F\,dA$$

where F is a smooth vector field and $\hat{n}$ is the unit normal vector to the curve ∂D. (Compare with the divergence theorem.)