Mini Problems 15

1. Let $\{x_n\}_{n\geq 1}$ be a sequence in $\mathbb{R}^2$ that converges to some point. Prove that the set $\{x_n\}_{n\geq 1}$ is Jordan measurable, with measure 0.

2. Suppose that you have sets $S \subseteq T \subseteq \mathbb{R}^n$ (where $n = 1$ or 2, or higher; it doesn't matter) and that you know T is Jordan measurable with $m(T) = 0$. Show that S is *also Jordan measurable,* and that $m(S) = 0$ as well. This can be done using nothing but the definition of Jordan measure. In particular, this implies the following fact: If $S \subseteq \mathbb{R}$ is *any set,* then $S \times \{0\}$ is Jordan measurable with measure 0 as a subset of $\mathbb{R}^2$, even though S might not be Jordan measurable in $\mathbb{R}$.

3. Prove that the set of integers $\mathbb{Z} \subseteq \mathbb{R}$ is not Jordan measurable, directly from the definition. Find an example of a bounded subset of $\mathbb{R}$ which is not Jordan measurable (if you are stuck, such an example can be found in the class notes).

4. Let $S \subseteq [a, b]$ be some set. Prove that if $\int_a^b \chi_S$ exists and is equal to 0 then S is Jordan measurable with measure 0.