Mini-Problems 1

1. Let $f : \mathbb{R} \to \mathbb{R}$ be a function. Show that the function $g : \mathbb{R} \to \mathbb{R}^2$ defined by $g(x) = (x, f(x))$ is always injective. The range of g is an object that you have seen before. What is it? (Hint: try sketching the range of g for a simple choice of f like $f(x) = x + 1$.)

2. If $f : X \to Y$ and $g : Y \to Z$ are functions, then their composite is $g \circ f : X \to Z$. Suppose that B is a subset of Z. Prove that $(g \circ f)^{-1}(B) = f^{-1}(g^{-1}(B))$.

3. If $f(x)$ and $g(x)$ are surjective functions from $\mathbb{R}$ to $\mathbb{R}$, does the function $(f(x), g(x))$ from $\mathbb{R}$ to $\mathbb{R}^2$ need to be surjective? Either prove that this is so, or find a counterexample.

4. Let $f : X \to Y$ be a function. **(i)** If B is a subset of Y, prove that $f^{-1}(B^c) = f^{-1}(B)^c$. **(ii)** Let A be a subset of X. Prove that if f is surjective then $f(A^c) \supseteq f(A)^c$ and that if f is injective then $f(A^c) \subseteq f(A)^c$. **(iii)** Take $X = Y = \mathbb{R}$, $A = [0, \infty)$ and let $f : X \to Y$ be the function $f(x) = x^2$. Calculate $f(A^c)$ and $f(A)^c$ and observe that neither is contained in the other.